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We propose a scheme for implementation of logical gates in a trapped ion inside a high-Q cavity. The ion is simultaneously interacting with a (classical) laser field as well as with the (quantized) cavity field. We demonstrate that simply by tuning the ionic internal levels with the frequencies of the fields, it is possible to construct a controlled-NOT gate in a three step procedure, having the ion's internal as well as motional levels as qubits. The cavity field is used as an auxiliary qubit and basically remains in the vacuum state.

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The coherent manipulation of simple quantum simples has become increasingly important for both the fundamental physics involved and prospective applications, especially on quantum information processing. Entanglement between two or more subsystems is normally required in order to have conditions for "quantum logical" operations to be performed. Two-level systems are natural candidates for building quantum bits (qubits), which are the elementary units for quantum information processing. As quantum subsystems which have shown themselves suitable for manipulation we may mention single ions interacting with laser fields [1], atoms and field modes inside high-Q cavities [2], and in molecules via NMR [4], for instance. Regarding the atoms (or ions), both internal (electronic) as well as vibrational motion states may be readily used for performing quantum operations, e.g., a controlled-NOT gate [1], and a phase gate [2]. It would be therefore interesting to explore other combinations of physical systems experimentally available. An interesting arrangement is a single trapped ion inside a cavity. The quantized field couples to the oscillating ion so that we have three quantum subsystems: the center-of-mass ionic oscillation, the ion's internal degrees of freedom, and the cavity field mode. A few papers may be found, in which it is investigated the influence of the field statistics on the ion dynamics [5,6], the transfer of coherence between the motional states and light [7], as well as a scheme to generate Bell-type states of the cavity-field and the vibrational motion [8]. On the experimental side, a single trapped ion has been succesfully used as a probe for the cavity field [9]. This of course opens up new possibilities for both trapped ion dynamics and cavity QED, and perhaps also for quantum information processing [10]. More recently, we also find articles discussing the combination of trapped ions with cavity QED [11,12].

In this paper we present a scheme for quantum state manipulation from which we may construct a quantum phase gate and a Hadamard gate, simply by adjusting atom-field detunings within the same experimental set-up, making possible to implement a controlled-NOT gate. A controlled-NOT gate transforms two-qubit states in the following way

$$
\begin{aligned}
|0,0\rangle &\Longrightarrow |0,0\rangle \\
|0,1\rangle &\Longrightarrow |0,1\rangle \\
|1,0\rangle &\Longrightarrow |1,1\rangle \\
|1,1\rangle &\Longrightarrow |1,0\rangle,
\end{aligned} \tag{1}
$$

i.e., the second qubit (target qubit) undergoes a change only if the first qubit (control qubit) is one, no changes occurring if its value is zero. Our scheme is based on the arrangement in which a single ion is trapped inside a cavity. The ion is coupled to both the cavity field as well as to a classical driving field. We also need an auxiliary state in order to perform the operations above. For instance, in the method presented in reference [13], other internal atomic states are used as auxiliary qubits. In our scheme, the two internal ionic levels represent the target qubit, and the ionic center-of-mass oscillation the control qubit, but differently from other schemes, here the quantized field states constitute the auxiliar qubit necessary for the implementation of a controlled-NOT logic gate.

It is not difficult to show that a controlled-NOT gate is equivalent to the application of a Hadamard gate, followed by the application of a phase gate, and an application of a Hadamard gate again. A Hadamard gate has the action

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$$
\begin{aligned}
|0\rangle &\Longrightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}} \\
|1\rangle &\Longrightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}},
\end{aligned} \tag{2}
$$

and a phase gate (causing a particular phase shift of π) is such that

$$
\begin{aligned}
|0,0\rangle &\Longrightarrow |0,0\rangle \\
|0,1\rangle &\Longrightarrow |0,1\rangle \\
|1,0\rangle &\Longrightarrow |1,0\rangle \\
|1,1\rangle &\Longrightarrow -|1,1\rangle.
\end{aligned} \tag{3}
$$

We shall seek for convenient interactions which would allow the implementation of the sequence of operations described above, bearing in mind the importance of the speed of operation of the logic gate, specially because of the unwanted action of the environement, that normally causes decoherence.

We consider the interaction of a (two-level) trapped ion in interaction with an external laser field (having frequency ω_L), as well as with a cavity field (having frequency ω_c). The hamiltonian for such a system reads

$$
\begin{aligned}
\hat{H} = \hbar\nu\hat{a}^\dagger\hat{a} + \hbar\omega_c\hat{b}^\dagger\hat{b} + \frac{\hbar\omega_0}{2}\sigma_z + \hbar\Omega\sigma_+ \exp\left[i\eta_L(\hat{a}^\dagger + \hat{a}) - i\omega_L t\right] + \\
+ \hbar\Omega\sigma_- \exp\left[-i\eta_L(\hat{a}^\dagger + \hat{a}) + i\omega_L t\right] + \hbar g(\sigma_+ + \sigma_-)(\hat{b}^\dagger + \hat{b}) \sin\eta_c(\hat{a}^\dagger + \hat{a}),
\end{aligned} \tag{4}
$$

where $\hat{a}^\dagger(\hat{a})$ are the creation (annihilation) operators relative to the vibrational motion excitations, $\hat{b}^\dagger(\hat{b})$ are the creation (annihilation) operators of the cavity field excitations, $\sigma_+(\sigma_-)$ are the raising (lowering) atomic operators, ω_0 is the atomic frequency, ν is the ionic vibrational frequency, and η_L, η_c are the Lamb-Dicke parameters relative to the laser field and the cavity field, respectively. We are going to suppose that we have a "double" Lamb-Dicke regime, or $\eta_L \ll 1$ and $\eta_c \ll 1$, so that we may write $\exp\left[i\eta_L(\hat{a}^\dagger + \hat{a})\right] \approx 1 + i\eta_L(\hat{a}^\dagger + \hat{a})$ and $\sin\eta_c(\hat{a}^\dagger + \hat{a}) \approx \eta_c(\hat{a}^\dagger + \hat{a})$. Under this approximation, the interaction hamiltonian in the interaction picture will be

$$
\begin{aligned}
\hat{H}_i = \hbar\Omega[\sigma_+ \exp(i\delta_{aL}t) + h.c.] + i\eta_L\hbar\Omega\left[\sigma_+\hat{a}\exp\{i\delta_{aL} - \nu)t\} - h.c.\right] + \\
+ i\eta_L\hbar\Omega\left[\sigma_+\hat{a}^\dagger\exp\{i\delta_{aL} + \nu)t\} - h.c.\right] + \eta_c\hbar g\left[\sigma_+\hat{a}^\dagger\hat{b}^\dagger\exp\{i\delta_{ac} + \nu + 2\omega_c)t\} + h.c.\right] + \\
+ \eta_c\hbar g\left[\sigma_+\hat{a}\hat{b}^\dagger\exp\{i\delta_{ac} - \nu + 2\omega_c)t\} + h.c.\right] + \eta_c\hbar g\left[\sigma_+\hat{a}\hat{b}\exp\{i\delta_{ac} - \nu)t\} + h.c.\right] + \\
+ \eta_c\hbar g\left[\sigma_+\hat{a}^\dagger\hat{b}\exp\{i\delta_{ac} + \nu + 2\omega_c)t\} + h.c.\right],
\end{aligned} \tag{5}
$$

where $\delta_{aL} = \omega_0 - \omega_L$ and $\delta_{ac} = \omega_0 - \omega_c$.

In order to implement the controlled-NOT gate in the system considered here, it is required the following: for the Hadamard gate [see Eq. (2)], we need an interaction hamiltonian of the type

$$
\hat{H}_{Hg} = \hbar\Omega[\sigma_+ + \sigma_-], \tag{6}
$$

and an application of a $\pi/4$ laser pulse. For the phase gate [see Eq. (3)], it is needed the following interaction hamiltonian

$$
\hat{H}_{pg} = \hbar\eta g[\sigma_+\hat{a}^\dagger\hat{b} + \sigma_-\hat{a}\hat{b}^\dagger], \tag{7}
$$

and a application of a π laser pulse.

For instance, the action of the phase gate above is such that

$$
\begin{aligned}
[|0\rangle_v|g\rangle]\,|0\rangle_f &\longrightarrow [|0\rangle_v|g\rangle]\,|0\rangle_f \\
[|0\rangle_v|e\rangle]\,|0\rangle_f &\longrightarrow [|0\rangle_v|e\rangle]\,|0\rangle_f \\
[|1\rangle_v|g\rangle]\,|0\rangle_f &\longrightarrow [|1\rangle_v|g\rangle]\,|0\rangle_f \\
[|1\rangle_v|e\rangle]\,|0\rangle_f &\longrightarrow -\,[|1\rangle_v|e\rangle]\,|0\rangle_f.
\end{aligned} \tag{8}
$$

It is required simply a cavity field (auxiliar qubit) initially prepared in the vacuum state. After a logical operation takes place, (e.g., after a π pulse) the cavity field remains in the vacuum state, and it is left ready for newcoming operations, as we see from (8). The system described by the hamiltonian in Eq. (5) makes possible the implementation of a Hadamard gate and a phase gate simply by independently tuning the atomic levels relatively to the cavity and laser fields. If $\delta_{aL} = 0$ (or $\omega_0 = \omega_L$) in (5) (after applying the rotating wave approximation), we end up with the hamiltonian necessary for the implementation of a Hadamard gate [see Eq. (6)]. On the other hand, if $\delta_{ac} = \omega_0 - \omega_c = -\nu$, the obtained hamiltonian will be precisely the one needed for the phase gate operation [see Eq. (7)]. Since it is necessary the sequential application of one Hadamard gate plus a phase gate and one Hadamard gate again to have a controlled-NOT gate, there is need of rapidly switching from the interaction hamiltonian in Eq. (6) (after applying a $\pi/4$ pulse), to the one in Eq. (7) and back to (6). This may be accomplished by applying static electric fields in order to tune the atomic energy levels either with the laser field or the cavity field.

A question that normally arises is the effect of decoherence on the gate operation. Here we have sources of decoherence coming from the trapped ion motion as well as the cavity field, that will affect the unitary evolution needed for quantum logical operations. However, in a state-of-the-art high-Q cavity, the typical decay time is of the order of $\tau_c \approx 0.2$s [14], while the ion's motional decoherence time is around $\tau_i \approx 1$ms [1]. It typically takes $T \approx \mu$s to perform two Hadamard and one phase gate operations [1], i.e., many operations are allowed within a virtually decoherence-free time-scale, which is an essential requisite for performing quantum computation.

We have presented an alternative scheme that would allow the implementation of quantum logical operations in cold trapped ions. The ion is supposed to be placed inside a high-Q cavity so that the gate operation is assisted by the (quantized) cavity field. A controlled-NOT gate may be constructed by applying laser pulses during convenient times, and also tuning the ion's internal levels either with the cavity field or with an external (classical) field. The cavity field remains most of the time in its vacuum state during the logical operations, which is desirable if one wants to avoid the destructive effects of cavity losses. It therefore seems that the system trapped ion + cavity represents an important system for quantum information processing.

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